Rendering of Transparent Objects in Polygon-Based Computer Holography

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Abstract: A novel technique is proposed for rendering transparent object in the polygon-based method. Refraction of light is simulated by the field propagation in the wavelength inside the object. Rotational transform of wave fields plays an important role to obtain the wave field at the boundary surface of the object. Simulated reconstruction of a computer-generated hologram calculated by the propose technique is demonstrated for verifying the validity.

1. Introduction

Computer holography is a 3D imaging technique that does not cause any vergence-accommodation conflict, unlike conventional 3D displays. We have proposed the polygon-based method and created large-scale computer-generated holograms (CGH) using the technique, whose number of pixels is over 4 billion [1-3]. In this method, the object is considered to be composed of many surface light sources, unlike conventional point-based techniques. This method features short computation time without any hardware assist. Recently, we proposed several rendering techniques of the polygon-based CGH, such as rendering of flat and smooth specular surfaces [2,3]. In this paper, we propose a novel technique to render transparent objects.



Fig. 1. Coordinate systems used in the polygonbased method.



Fig. 2. Restrictions of the object shape in the proposed technique.

2. Rendering of refraction light in Polygon-Based CGH

Figure 1 shows the three types of the coordinates systems used in the polygon-based method; those are hologram coordinates $(\hat{x}, \hat{y}, \hat{z})$, parallel local coordinates $(\hat{x}_n, \hat{y}_n, \hat{z}_n)$ and tilted local coordinates (x_n, y_n, z_n) of polygon *n*. The hologram is in the $(\hat{x}, \hat{y}, 0)$ plane of the hologram coordinates. Two local coordinates are defined for each polygon and share the origin, but these are tilted so that the polygon face is in the $(x_n, y_n, 0)$ plane.

The 3D object rendered by the proposed technique has several restrictions, as shown in Fig.2. The back face of the object must be a single planar polygon placed at $\hat{z} = \hat{z}_{obj}$, which is parallel to the hologram. The object must not have concave. The cross-section must take the maximum at back face P₀, and furthermore the closer section to the hologram must have smaller cross-section in $\hat{z} > \hat{z}_{obj}$.

The wave field $f(\hat{x}, \hat{y}; \hat{z}_{obi})$ entering into the back face P₀ is divided into inside and outside wave fields as follows.

$$\begin{aligned} f_{inside}(\hat{x}, \hat{y}; \hat{z}_{obj}) &= f(\hat{x}, \hat{y}; \hat{z}_{obj}) \times A_0(\hat{x}, \hat{y}), \\ f_{outside}(\hat{x}, \hat{y}; \hat{z}_{obi}) &= f(\hat{x}, \hat{y}; \hat{z}_{obi}) \times \{1 - A_0(\hat{x}, \hat{y})\}, \end{aligned}$$
(1)

where $A_0(\hat{x}, \hat{y})$ is a binary mask. The value is 1 inside P_0 or 0 outside P_0 . The outside wave field is propagated to the hologram plane in wavelength λ_0 of the outside media, which is commonly the same as the vacuum wavelength. The inside wave field is propagated to the position of each polygon in wavelength $\lambda_1 = \lambda_0 / n_{obj}$, where n_{obj} is the refractive index inside the object.

Figure 3 shows the procedure for calculation of the wave field propagating through the inside media onto polygon P_n . The wave field $f_n(\hat{x}_n, \hat{y}_n; \lambda_1)$ in the $(\hat{x}_n, \hat{y}_n, 0)$ plane of P_n , as in Fig.3(a), is converted into the field in the polygon face by using rotational transform [4].



wave field passing through polygon

 $f_n(x_n, y_n; \lambda_1) = \mathbf{R} \left\{ f_n(\hat{x}_n, \hat{y}_n; \lambda_1) \right\},\tag{2}$

where $R\{\cdot\}$ stands for the rotational transform and $f_n(x_n, y_n; \lambda_1)$ is the wave field in the $(x_n, y_n, 0)$ plane of tilted local coordinates, as in (b). Here, a technique called spectrum remapping [1-3] is used to reduce computation time. The wave field in the polygon face is masked as in (c) in order to restrict the field inside the polygon as follows,

$$f'_{n}(x_{n}, y_{n}; \lambda_{1}) = f_{n}(x_{n}, y_{n}; \lambda_{1}) \times A_{n}(x_{n}, y_{n}), \qquad (3)$$

where $A_n(x_n, y_n)$ is the binary mask of P_n . When the field crosses the boundary after masking, the wavelength of the wave field changes from λ_1 to λ_0 . Then, the wave field is again transformed rotationally to obtain the field in parallel local coordinates as in (d), i.e.,

$$f'_{n}(\hat{x}_{n}, \hat{y}_{n}; \lambda_{0}) = \mathbf{R}^{-1} \left\{ f'_{n}(x_{n}, y_{n}; \lambda_{0}) \right\},$$
(4)

where $\mathbf{R}^{-1}\{\cdot\}$ stands for inverse rotational transform. Here not that the outside wavelength is used in this transform. This field is propagated to the hologram plane. Finally, all fields are superposed in the hologram plane as follows,

$$f_{holo}(\hat{x}, \hat{y}, 0) = \sum_{n} f_{n}(\hat{x}, \hat{y}, 0), \qquad (5)$$

where $f_n(\hat{x}, \hat{y}, 0)$ is the wave field in the hologram plane, which passes the polygon P_n.

3. Simulation

 \mathbf{P}_n .

The CGH of the 3D scene shown in Fig.4 was calculated and reconstructed by simulation using virtual optics [5]. The parameters used are summarized in Table 1. The Teapot model is cut in half and arranged in contact with a checkered wall. The simulated reconstruction of this CGH is shown Fig. 5.

Table 1. Parameters used	for simulation.
Number of pixels $(N_x \times N_y)$	32,768 × 32,768
Pixel pitches	$1\mu m \times 1\mu m$
Refraction index of object	1.33
Reconstruction wavelength	632.8 nm

4. Conclusion

We proposed a new technique to render transparent objects in the polygon-based method. We calculated a CGH by using the technique. Simulated reconstruction of the CGH confirms the validity of the proposed technique.

This work was supported by JSPS KAKENHI (15K00512) and MEXT Strategic Research Foundation at Private Universities (2013-2017).

References

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Fig. 4. The 3D scene of the CGH.



Fig. 5. Simulated reconstruction of the CGH rendered by the proposed technique.

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