

New Creation Algorithm for Digitally Synthesized Holograms in Surface Model by Diffraction from Tilted Planes

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ABSTRACT

Novel method for synthesizing light waves from objects expressed in surface model is presented for fast creation of digitally synthetic holograms. This method has a feature of coordinates rotation in Fourier domain, in which only twice FFTs and an interpolation of spectrum are necessary for calculation of light waves from each object's plane. Therefore, presented method is rather faster than conventional ray-tracing when large scale full-parallax holograms are synthesized in a surface model. Phases of a surface object are also discussed in relation to controlling the direction and diffusiveness of object waves. Furthermore, fabrication of a hologram based on the method and its reconstruction are presented for a demonstration.

Keywords: Computer-generated Holograms, Digital hologram, Angular Spectrum of Plane Waves, Surface Model

1. INTRODUCTION

Computer-Generated Holograms or digitally synthetic holograms are desired media to create three-dimensional (3-D) autostereoscopic images of virtual objects. However, the technology has several major problems should be solved. One of them is to need very high spatial resolution to fabricate the holograms. Recent electron-beam lithography techniques enable us to fabricate synthetic holograms with sub micrometer structures.¹ In such large-scale holograms, a computational explosion in the numerical calculation of complex amplitude distributions for 3-D objects causes another problem.

Recently, a ray-tracing method^{2,3} is commonly used to generate the complex amplitudes of light waves emitted from 3-D objects, in which objects are composed of a large number of point sources of light, and complex amplitudes of spherical waves from each point source is superimposed on the hologram plane. The ray-tracing method is simple in principle and is potentially the most flexible in synthesizing holograms for true 3-D objects. The method, however, requires an enormous computation time for generating the complex amplitudes or the fringe patterns, especially in full-parallax holograms. A typical method for reducing computation time in ray tracing is to calculate the real-valued fringe intensity instead of the complex-valued amplitude of the light wave from objects.^{3,4} There are also other methods using geometric symmetry to avoid redundant calculation⁵ and lookup table⁴ that remarkably improves the computation time. Some arithmetic acceleration algorithms such as difference formulas⁶ and recurrence formulas⁷ has been presented for accelerating the computation. A computer-graphics hardware is also used to assist in calculating the diffraction pattern of point sources.⁸ Moreover, a special purpose computer for fast synthesis of 3-D holograms is another attractive approach.^{9,10}

In general, computation time in ray-tracing is proportional to both number of sampling on the hologram and point sources on the objects. Therefore, the computation time can be written as⁷

$$T_{\text{total}} = \tau_a MN, \quad (1)$$

where M and N is number of pixels of a hologram and point sources of an object, respectively. τ_a is, here, an elemental time coefficient that depends on the processors, methods, algorithms and so on. The coefficient is

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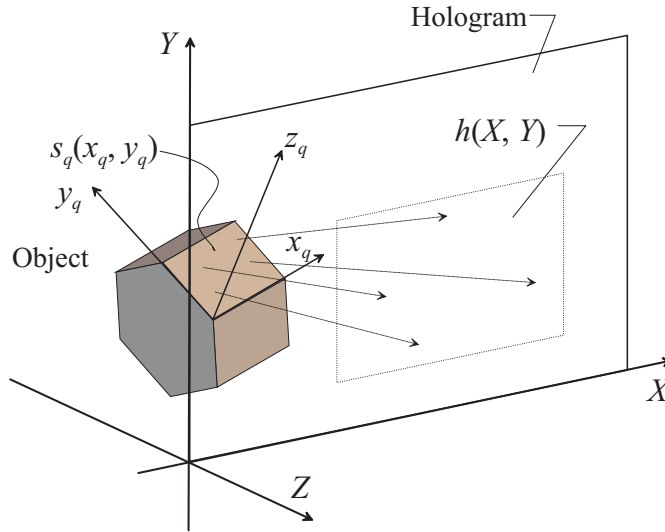


Figure 1: An object model and coordinate system used for synthesizing object waves.

defined as the time necessary for calculating the fringe intensity of a single point source in a single pixel. τ_a is, for example, 47 ns with the recurrence formula on an Alpha 21164A (600 MHz) processor,⁷ or 0.6 ns with the difference formula on Pentium II (450 MHz)*. Suppose that we synthesize a hologram of which dimensions are $5\text{cm} \times 5\text{cm}$. If the hologram would be sampled with interval of $1\text{ }\mu\text{m}$ and the objects have 10^5 point sources, its computation time by the difference formula is approximately 40 hours in a rough estimation. Processor performances are continuously increasing year by year, and new technology such as parallel processing will be introduced into synthesis of digital holograms in the future. However, it seems to be hard to get over the essential drawbacks of ray tracing in the point source model.

Another object model to synthesize object waves has been suggested in a Fresnel hologram.¹¹ In the model, an object is composed of several small planar segments, and waves diffracted by the planar segments are superimposed on a hologram plane. To implement this model, numerical algorithm to calculate diffraction by tilted apertures is necessary for synthesizing object waves. In near field diffraction, a similar scheme has been suggested by a method based on the angular spectrum of plane waves,¹² in which an idea on coordinates rotation in the Fourier domain has been mentioned briefly. However, any numerical and experimental results as well as complete formulation are not provided in the literature. We have independently developed algorithm based on the angular spectrum of plane waves to simulate optical diffraction by tilted apertures as well as diffraction onto tilted screens.¹³ By use of this method, we can calculate complex amplitudes of light waves emitted from objects structured upon the plane surface model.

2. OBJECT MODEL

Coordinates system and geometry used in this paper is shown in Fig. 1. The global coordinates is denoted by (X, Y, Z) , and the hologram is placed on the $(X, Y, 0)$ plane. Objects composed of plane surfaces are usually positioned in $Z < 0$. Each surface of the object, specified by q , possesses its own local coordinates (x_q, y_q, z_q) , in which the surface is placed on the $(x_q, y_q, 0)$ plane. Distribution of complex amplitudes $s_q(x_q, y_q)$, referred to as the surface property function in this paper, is defined on the $(x_q, y_q, 0)$ plane for each surface segment. Each surface property function contains information on the surface segment such as the shape, brightness, diffusiveness, feel of material, textures and so on. To obtain distribution of complex amplitudes $H(X, Y)$ of object waves on the hologram, light waves $h_q(X, Y)$ diffracted by each surface property function are calculated

*We estimate the coefficient τ_a for the difference formula from computation time reported in Ref. 6

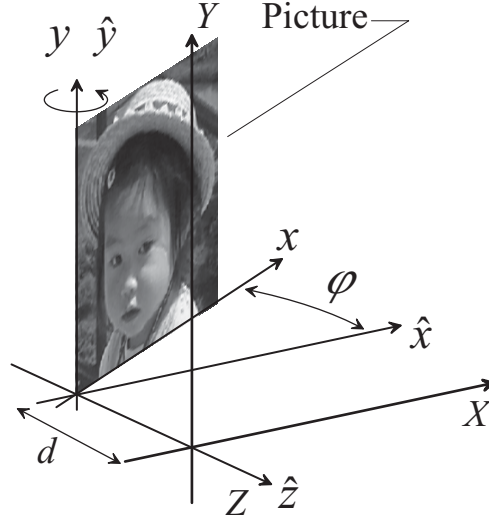


Figure 2: Geometry in single-axis rotation.

by the method described in the following section, and then they are superimposed on the hologram plane as follows:

$$H(X, Y) = \sum_q h_q(X, Y). \quad (2)$$

Two steps are necessary to calculate individual light waves emitted from each surface segment; local coordinates (x_q, y_q, z_q) is made to rotate so as to be parallel to the hologram, and then object waves on the hologram are calculated from the rotated complex amplitudes. In the Appendix A and the following section we summarize the step for rotation of local coordinates.

3. SINGLE AXIS ROTATION

In conventional simulation methods for diffraction such as the Fresnel-Kirchhoff formula, its Fresnel and Fraunhofer approximation, and ordinary angular spectrum of plane waves,¹⁴ a common constraint is that a screen must be parallel to the aperture. Therefore, object waves $h_q(X, Y)$ on the hologram can not be given by means of these methods. We derived general formulation for numerical simulation in diffraction by tilted apertures¹³ and summarize it in the appendix A.

In this Section and the Appendix A, (x, y, z) denotes local coordinates not parallel to global coordinates, and $(\hat{x}, \hat{y}, \hat{z})$ denotes another coordinates locally defined, which shares its origin with (x, y, z) coordinates but is parallel to global coordinates. This local coordinates is referred to as rotated local coordinates.

3.1. Formulation and Performance

Object waves emitted from plane surface rotated upon single-axis was synthesized and its hologram reconstructing the 3-D images was fabricated to verify and demonstrate the method. In this experiment, planes composing an object are rotated on \hat{y} axis with angle of φ , and origin of the rotated local coordinates is positioned on $(0, 0, -d)$ in the global coordinates as shown in Fig. 2.

Transformation matrix for the single-axis rotation is given as

$$\mathbf{T}^{-1} = \begin{bmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{bmatrix}. \quad (3)$$

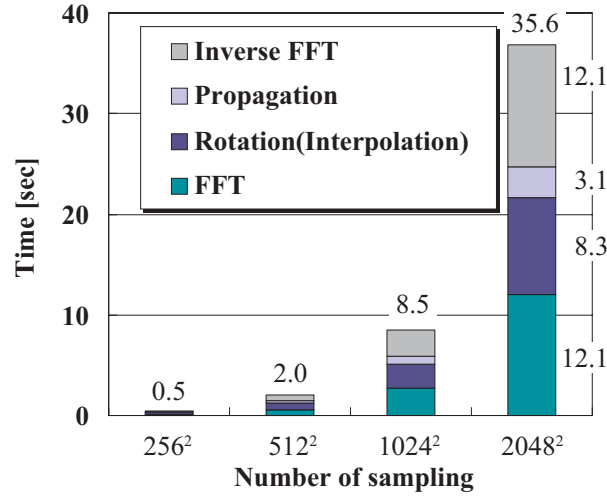


Figure 3: Performance of the single axis rotation accompanied with translational propagation.

By substituting elements of the matrix (3) into Eqs. (10) and (13), complex amplitudes on the rotated plane are given by the inverse Fourier transformation as follows:

$$h(\hat{x}, \hat{y}) = \mathcal{F}^{-1} \{ \hat{T}(\hat{u} \cos \varphi + \hat{w}(\hat{u}, \hat{v}) \sin \varphi, \hat{v}) | \cos \varphi + \frac{\hat{u}}{\hat{w}(\hat{u}, \hat{v})} \sin \varphi | \}. \quad (4)$$

Performances of the single-axis rotation, measured on a Pentium III (1 GHz) processor, are shown in Fig. 3. The FFT and inverse FFT consume approximately 68 % of processor time, and therefore total computation time as a function of number of sampling points approximately agrees with that of FFT.

3.2. Phases of Surface Property Functions

In this section, we discuss single-axis rotation of a two-dimensional picture, i.e. amplitudes of a picture is set to that of the surface property function. Question in the case is how to determine its phase. When a constant phase is given to the picture, light waves emitted from the picture propagate in a direction perpendicular to the picture surface, and therefore, in cases of large φ or d , the object waves may not reach to the hologram plane. This is shown in the row (1) of Fig. 4. Amplitudes and phases on the hologram plane in $d = 0$ are shown in column (a) and (b), respectively. When $d = 0$, the amplitude image of the picture is approximately placed at the center of the hologram. The amplitude image is a little unclear only around the right edge of the picture because of optical diffraction. However, the spectrum (c) has a sharp spot and the spot shifts from the center. As a result, when $d > 0$, the amplitude image in (d) is not only unclearer than in (a) but also shift right.

In the row (2) of Fig. 4, phase of a plane wave, which propagates along Z axis, is set to the phase of the picture object. By this linear phase, the light emitted from the picture irradiates the hologram perpendicularly. Accordingly, the amplitude image keeps the center of the hologram, as shown in column (d).

Direction of object waves can be controlled by the linear phase, but the spectrum still keeps a sharp spot, which means that the object wave is hardly diffracted by the surface object and therefore does not reach to the whole of the hologram. This is attributed to deficiency of diffusiveness of the surface. Randomizing the object phase can achieve increase in diffusiveness, but it also causes speckle noise. For numerical diffusion of light without speckle noise a special phase called the digital diffuser has been suggested in Fourier hologram.¹⁵ The row (3) of Fig. 4 shows results of superimposing the digital diffuser on the linear phase. The object is much more diffusive than that only with the linear phase. As a result, the original picture image disappears in amplitude images on the hologram because of strong diffraction.

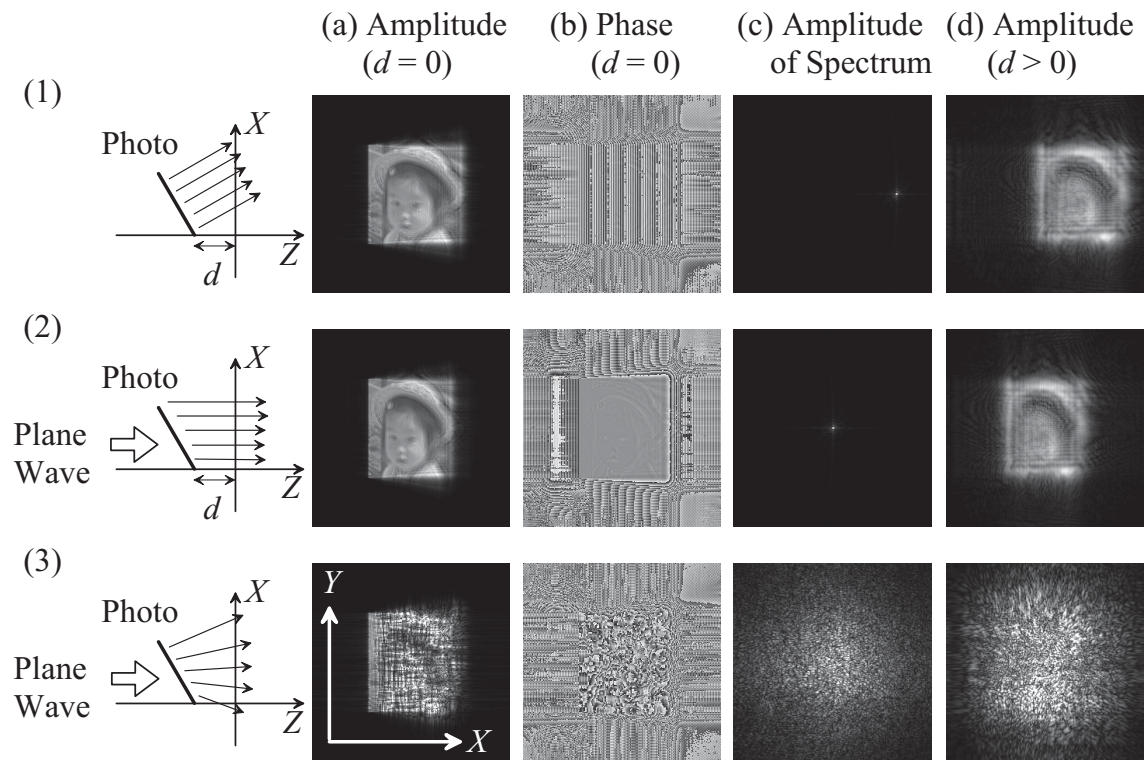


Figure 4. Phases for a picture object; (1) a constant phase, (2) a linear phase, (3) superimposition of a digital diffuser on the linear phase.

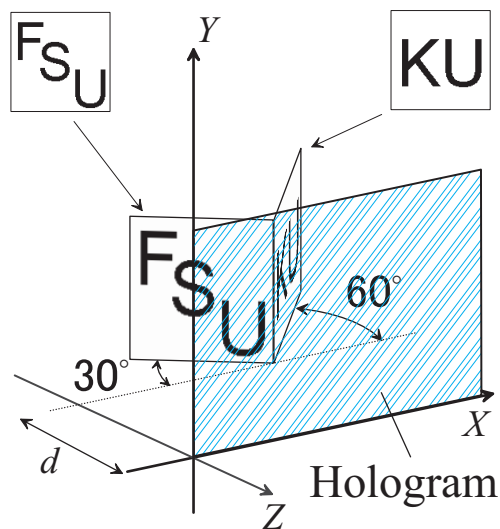


Figure 5: An object numerically constructed for fabricating a hologram.

4. FABRICATION AND RECONSTRUCTION OF A HOLOGRAM FOR A 3-D SURFACE OBJECT

For verifying the method described in above sections, an object wave is synthesized from surface planes with a linear and diffused phase in a setup for the single-axis rotation. The object numerically constructed is shown in Fig. 5. The object is composed of two tilted plane surfaces, which have binary images including the letter ‘FSU’ and ‘KU’ as their amplitude distribution and the linear and diffused phase as their phase distribution. Parameters used for synthesizing the object wave and for fabricating its hologram are summarized in Table 1.

Table 1: Parameters used for synthesizing an object wave and fabricating a hologram.

Surface property function	
Number of sampling	8192×4096
Sampling pitch	$2\mu\text{m} \times 4\mu\text{m}$
Distance (d)	60 mm
Phase	linear & Bräuer digital diffuser
Hologram	
Pixels	16384×4096
Pixel size	$2\mu\text{m} \times 4\mu\text{m}$
Type	Binary amplitude hologram
Coding	Point oriented coding by simple threshold
Reference wave	Plane wave with incidence angle of 4°

The hologram was fabricated by a special laser printer originally developed for printing holograms. The dimensions of the hologram is $32.8 \text{ mm} \times 16.4 \text{ mm}$ and its viewing zone is $18^\circ \times 9^\circ$ according to the pixel size. An optically reconstructed image of the fabricated hologram is shown in Fig. 6.

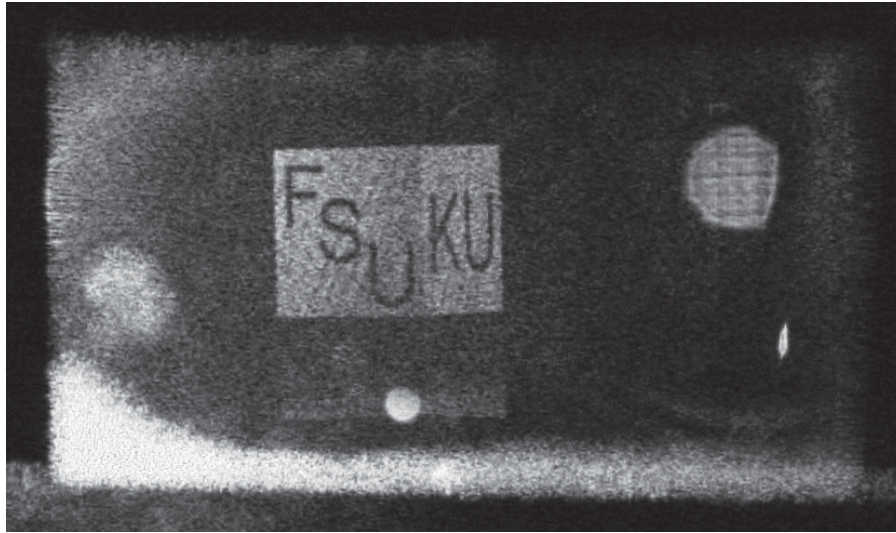


Figure 6. Photograph of optical reconstruction of a fabricated hologram. The object is composed of two plane surfaces not parallel to the hologram.

5. CONCLUSION

A novel method for calculating complex amplitudes of light wave from virtual 3-D objects was presented for fast creation of digitally synthetic full-parallax holograms. In most of synthetic 3-D holograms, objects are composed of point sources, and the spherical waves emitted from the point sources are superimposed on a hologram plane, whereas our concept has a feature that 3-D objects are composed of small planar surfaces and diffraction by the surfaces is simulated. For realization of the concept i.e. for effective calculation of optical diffraction by a tilted surface, we developed algorithm based on the angular spectrum of plane waves.

In the method, a complex function representing optical properties of a surface in the object is defined in local coordinate system. This surface property function includes information on the shape, diffusiveness and additional properties such as texture of the surface. The local coordinates is rotated by using the algorithm, so that the plane including the surface property function becomes parallel to the hologram. Finally the surface property function on the rotated plane is numerically diffracted onto the hologram by conventional method. Just twice FFTs and an interpolation of spectrum are necessary for the rotation step, and therefore computation can be faster than ray tracing of point sources.

APPENDIX A. SUMMARY OF GENRAL FORMULATION FOR COORDINATES ROTATION IN THE FOURIER DOMAIN

This section summarizes derivation of general formulation for coordinate rotation in the Fourier domain,¹³ based on the angular spectrum of plane waves.

Fourier spectrum of a surface property function is given as

$$T(u, v) = \mathcal{F}\{s(x, y)\} = \iint_{-\infty}^{\infty} s(x, y) \exp[-i2\pi(ux + vy)] dx dy. \quad (5)$$

where $\mathcal{F}\{s(x, y)\}$ denotes the Fourier transformation of $s(x, y)$. By physical interpretation of integrand in the inverse Fourier transformation of $T(u, v)$, the wave vector of plane waves can be associated with Fourier frequency on the $(x, y, 0)$ plane as follows:

$$\mathbf{k} = \frac{2\pi}{\lambda} [k_x \quad k_y \quad k_z], \quad (6)$$

$$= 2\pi [u \quad v \quad w(u, v)], \quad (7)$$

where the wave vector is defined as $|\mathbf{k}| = 2\pi/\lambda$ and $k_x^2 + k_y^2 + k_z^2 = 1$, and $w(u, v) = \sqrt{\lambda^{-2} - u^2 - v^2}$. There is the same relation in the rotated local coordinates. Therefore, when rotation matrix from (x, y, z) to $(\hat{x}, \hat{y}, \hat{z})$ is defined as

$$\hat{\mathbf{k}} = \mathbf{T}\mathbf{k}, \quad \mathbf{k} = \mathbf{T}^{-1}\hat{\mathbf{k}}, \quad (8)$$

$$\mathbf{T}^{-1} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}, \quad (9)$$

the Fourier frequencies (u, v) in local coordinates are given as functions of frequencies (\hat{u}, \hat{v}) in rotated local coordinates as follows:

$$\begin{aligned} u &= \alpha(\hat{u}, \hat{v}) = a_1\hat{u} + a_2\hat{v} + a_3\hat{w}(\hat{u}, \hat{v}), \\ v &= \beta(\hat{u}, \hat{v}) = a_4\hat{u} + a_5\hat{v} + a_6\hat{w}(\hat{u}, \hat{v}). \end{aligned} \quad (10)$$

Thus, the Fourier spectrum $\hat{T}(\hat{u}, \hat{v})$ in rotated coordinates is given by using the Fourier spectrum in local coordinates.

$$\hat{T}(\hat{u}, \hat{v}) = T(\alpha(\hat{u}, \hat{v}), \beta(\hat{u}, \hat{v})). \quad (11)$$

The complex amplitudes $h(\hat{x}, \hat{y})$ in rotated local coordinates is given by integration of the spectrum $\hat{T}(\hat{u}, \hat{v})$.

$$h(\hat{x}, \hat{y}) = \mathcal{F}^{-1}\{\hat{T}(\hat{u}, \hat{v})|J(\hat{u}, \hat{v})|\}, \quad (12)$$

where Jacobian $J(\hat{u}, \hat{v})$ defined as

$$\begin{aligned} J(\hat{u}, \hat{v}) &= \frac{\partial \alpha}{\partial \hat{u}} \frac{\partial \beta}{\partial \hat{v}} - \frac{\partial \alpha}{\partial \hat{v}} \frac{\partial \beta}{\partial \hat{u}} \\ &= (a_1 a_5 - a_2 a_4) + (a_2 a_6 - a_3 a_5) \frac{\hat{u}}{\hat{w}(\hat{u}, \hat{v})} + (a_3 a_4 - a_1 a_6) \frac{\hat{v}}{\hat{w}(\hat{u}, \hat{v})}, \end{aligned} \quad (13)$$

is necessary for energy conservation of the fields because of non-linearity of the rotational transformation of Eq. (10).

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