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Rigorous Light-Shielding for Hidden-Surface Removal in High-Definition Computer Holography

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Abstract—A technique is proposed for applying rigorous light-shielding to high-definition computer holography. This technique reduces occlusion errors caused in conventional silhouette masking that is currently used for hidden-surface removal in high-definition computer holography.

Keywords*-computer holography; computer-generated hologram; hidden surface removel*

I. INTRODUCTION

We have presented some high-definition computer-generated holograms (CGH) that are computed by polygon-based method [1, 2] and fabricated by laser lithography. Optical reconstruction of these high-definition CGHs is being comparable to that in traditional optical holography. In this high-definition computer holography, the technique of the silhouette masking plays an important role to shield light behind the object and prevent the reconstructed object from being see-through [3, 4]. This technique provides hidden surface removal like conventional computer graphics.

A drawback of the silhouette method is that the light is shielded by the silhouette-shaped mask in a plane parallel to the hologram. Therefore, if the CGH has very high physical resolution and thus large diffraction angles, off-axis lights cannot be shielded by the silhouette masks and causes occlusion errors that give unexpected black shadow or sometimes partial phantom images [4, 5]. To remove the occlusion errors in the silhouette masking, we have already proposed more rigorous method for light shielding [6]. In this technique, the wave-field is calculated in the same plane as the polygon by using the numerical technique of rotational transform of wave-fields [7, 8] and masked by using the polygon-shaped masks.

However, only the principle of this rigorous light-shielding technique has been verified in the literature. The technique has never been applied to the actual high-definition CGHs, because the rotational transform of the wave-field is very time-consuming in high-definition computer holography in that number of sampling points reaches to more than billions or sometimes more than ten billions [9, 10].

In this paper, we propose a new technique to apply the rigorous light-shielding method to high-definition computer holography. One of the techniques we propose in this paper is to clip the wave-field in order to reduce the sampling number in the rotational transform. Another one is to optimize the sampling area and interval of the wave-field in Fourier space. This is also for reducing the number of samplings of wave-field in rotational transform.

II. RIGOROUS LIGHT-SHIELDING

If the field is near paraxial, i.e. if the light propagates almost along the optical axis, the silhouette masking is a good trade-off between the computational cost and quality of the reconstructed image. However, if viewing zone of the hologram is large enough that the hologram reconstructs the auto-stereoscopic image, the field is no longer paraxial, and therefore occlusion errors are produced in the reconstructed image. This occlusion error can be removed by masking the background field with polygon itself in the plain in which the polygon is laid. To realize this processing, the background field, initially given in the plane parallel to the hologram, must be rotationally transformed once.

A. Rotational transform of wave fields

Suppose that the background wave-field $\hat{h}(\hat{x},\hat{y})$ is initially given in a plane $(\hat{x},\hat{y},\hat{z}=0)$ parallel to the hologram. The coordinates $(\hat{x},\hat{y},\hat{z})$ is referred to as parallel coordinates in this report. The basic rotational transform of the wave-field is written as

$$\begin{aligned} h(x,y) &= \mathrm{Rot}\left\{\hat{h}(\hat{x},\hat{y})\right\} \\ &= F^{-1}\left\{R\left\{F\{\hat{h}(\hat{x},\hat{y})\}\right\}\right\}, \end{aligned} \tag{1}$$

where the resultant field $h(x, y)$ is given in the plane $(x, y, z = 0)$ of other local coordinates that shares the origin with the parallel coordinates but the plane $(x, y, z = 0)$ is not parallel to the hologram. This is referred to as the tilted coordinates. Here, $F\{\cdot\}$ denote Fourier transform as follow:

$$\hat{H}(\hat{u},\hat{v}) = F\{\hat{h}(\hat{x},\hat{y})\} \tag{2}$$

where $(\hat{u},\hat{v})$ and (u,v) are spatial frequency in the parallel $(\hat{x},\hat{y},\hat{z}=0)$ and the tilted plane $(x,y,z=0)$, respectively.

Suppose that the transform matrix for coordinates rotation is defined as

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \mathbf{T} \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{T} = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix}, \tag{3}$$

the coordinates rotation in Fourier space $R\{\cdot\}$ is given as

$$\begin{aligned} & R\{\hat{H}(\hat{u},\hat{v})\} \\ & = \hat{H}(a_1 u + a_2 v + a_3 w(u,v), a_4 u + a_5 v + a_6 w(u,v)) \\ & = H(u,v) \end{aligned} \tag{4}$$

where $w(u, v)$ is spatial frequency in z direction perpendicular to the tilted plane and given by

$$w(u,v) = \sqrt{\lambda^{-2} - u^2 - v^2}. \tag{5}$$

Finally, the wave-field is given in the tilted plane by inverse Fourier transform as follow:

$$h(x, y) = F^{-1}\{H(u,v)\}. \tag{6}$$

B. *Masking fields in the tilted plane*

Fig. 1 shows the procedure for shielding the background field that should be shielded by a polygon. The mask function for the polygon must be multiplied by the background field given in the tilted plane of the polygon. Therefore, as in (a), the background field is obtained in the tilted plane from $\hat{h}(\hat{x},\hat{y})$ by rotational transform and then, as in (b), masked as follow:

$$h'(x, y) = \mathrm{Rot}\{\hat{h}(\hat{x},\hat{y})\} \times m(x, y), \tag{7}$$

where $m(x, y)$ is the binary mask function defined in the tilted plane, whose shape is the same as the polygon, i.e., the mask function is given as follow:

$$m(x, y) = \begin{cases} 1 & \text{inside polygon} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

The masked background field plane is calculated in the parallel by inverse rotational transform as in Fig. 1 (c):

$$\begin{aligned} \hat{h}'(\hat{x},\hat{y}) &= \mathrm{Rot}^{-1}\{h'(x, y)\} \\ &= \mathrm{Rot}^{-1}\left\{\mathrm{Rot}\{\hat{h}(\hat{x},\hat{y})\} \times m(x, y)\right\}. \end{aligned} \tag{9}$$

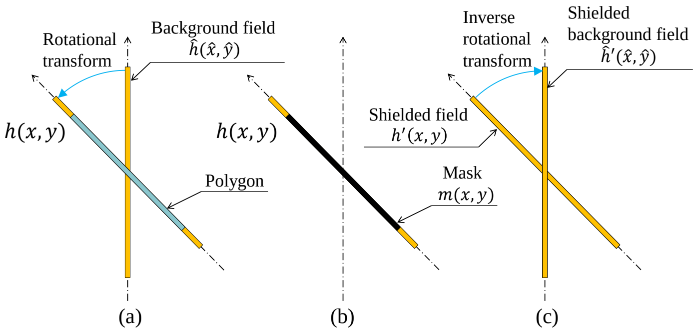

Figure 1. The procedure for rigorous light-sheilding.

III. REDUCTION OF COMPUTATIONAL COST

The procedure for the rigorous light-shielding is very time consuming, because it includes twice rotational transform of the background field whose number of samplings is more than billions in high-definition CGHs. To reduce the computational cost, we mainly adopt three techniques described as follow.

A. *Clipping the background field*

Polygons are commonly much smaller than the area of the background field, i.e. only a small part of the background field is shielded by a polygon. Therefore, the rotational transform should be performed in a small area instead of the whole background field, i.e. the background field should be clipped in the small area and then the field $h(x, y)$ in the tilted plane should be calculated by the rotational transform. Here, the required wave-field for the accurate light-shielding is a portion which enters into the polygon and is affected by light-shielding. We determine the required area in the whole background field by using the maximum diffraction area of the polygon [9], as in Fig. 2. Here, the maximum diffraction area is determined by vertex positions of the polygon and the maximum diffraction angle given by

$$\theta_{\max} = \sin^{-1}\left(\frac{\lambda}{2\Delta x}\right), \tag{10}$$

where Δx and λ are the sampling interval of the background field and the wavelength, respectively.

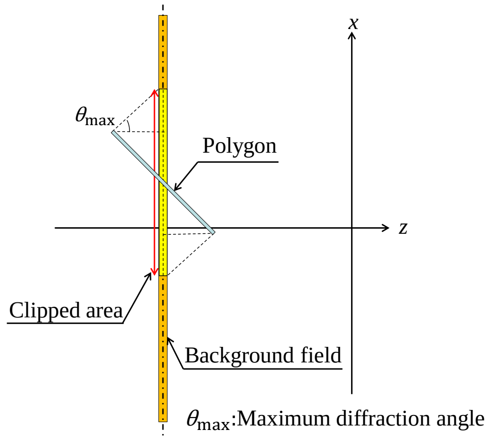

Figure 2. Clipping area of the background field.

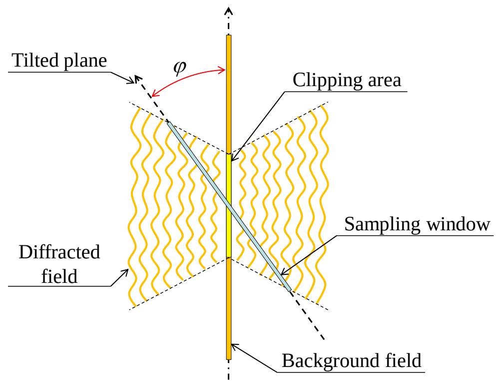

Figure 3. The sampling window required for avoiding information loss in the the tilted plane.

B. *Sampling reduction based on Babinet's principle*

As shown in Fig. 3, the sampling window in the tilted plane must include the whole field diffracted by the clipped area of the background field. This is necessary for avoiding information loss of the clipped field. However, the sampling window must be very large or sometimes infinity when polygons have a large tilt angle φ. To solve this problem, we use Babinet's principle and partial field propagation technique [11].

According to well-known Babinet's principle, light-shielding by the mask is equivalent to that by the aperture given by inverting the mask, as shown in Fig. 4. Here, the aperture function is given in the tilted plane as follow:

$$A(x,y) = 1 - m(x,y)\,. \tag{11}$$

When we use the aperture function $A(x,y)$ instead of the mask function $m(x,y)$ for masking the field, the sampling area required for light-shielding can be reduced only to the small portion of the wave-field, i.e. the area opened in the aperture whose shape is the same as the polygon. By using this technique, we can avoid the problem of infinity sampling window as shown in Fig. 3, because only the sampling window that cover the opened area of the aperture is necessary for light shielding.

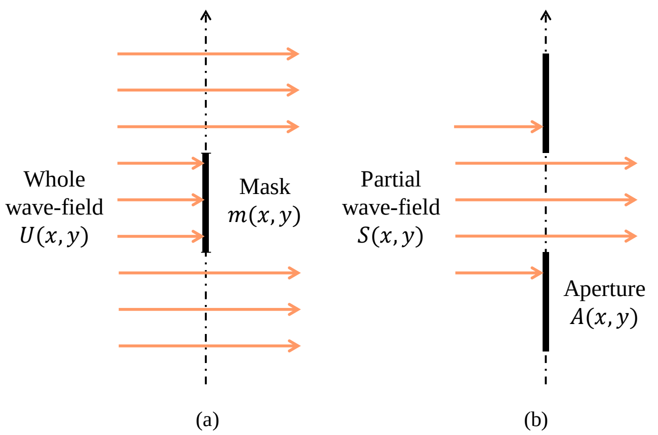

Figure 4. Light-shielding by the mask (a) and aperture (b). These can be equivalent to each other by using Babinet's principle and a partial field propgation [11]

C. *Optimizing the sampling window in Fourier space*

Figure 5 shows transition of sampling area in Fourier space for rotational transform. The sampling area initially forms a rectangle because of using FFT as in (a). However, as shown in (b), the sampling area of $H(u,v)$ is distorted after applying coordinates rotation R{} and positioned far from the origin. Since FFT do not work well in such cases, new shifted coordinates should be introduced into Fourier space as follow:

$$(\tilde{u},\tilde{v}) = (u - u_0, v - v_0), \tag{12}$$

where $(u_0,v_0) = (a_7/\lambda, a_8/\lambda)$. The inverse FFT is performed in the shifted coordinates and the field in the tilted plane is given by [7]

$$h(x,y) = F^{-1}\{\tilde{H}(\tilde{u},\tilde{v})\}\exp[i2\pi(u_0 x + v_0 y)]\,. \tag{13}$$

After multiplying the mask function we re-defined the masked field as $h'(x,y) = F^{-1}\{\tilde{H}(\tilde{u},\tilde{v})\} \times m(x,y)$. As a result, the Fourier transform for inverse rotation is written as

$$\begin{aligned} F\{h(x,y)m(x,y)\} &= F\{h'(x,y)\exp[i2\pi(u_0 x + v_0 y)]\} \\ &= H'(u+u_0, v+v_0), \end{aligned} \tag{14}$$

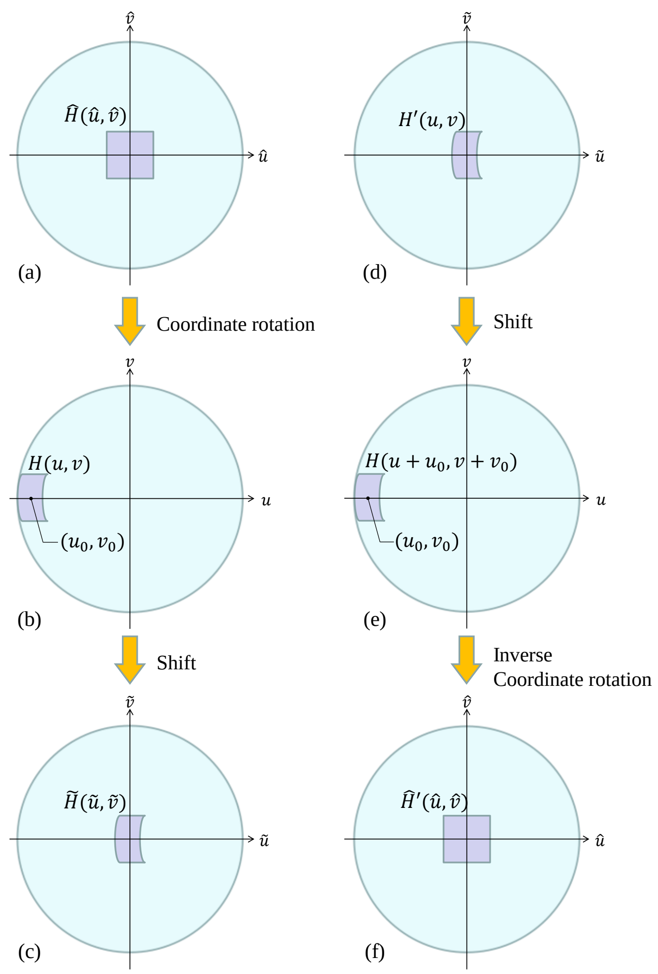

Figure 5. Transition of sampling area in Fourier apace.

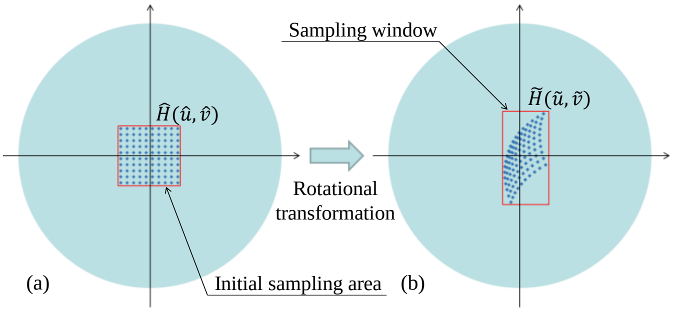

Figure 6. Optimization of sampling window in Fourier space.

where $H'(u,v) = F\{h'(x,y)\}$. Thus, the sampling area is again shifted from the origin, as shown in Fig. 5(e). Finally, the inverse coordinates rotation $R^{-1}\{\}$ restores the sampling area to a rectangular shape suited for FFT.

Through the entire processes of the rigorous light-shielding, the shape of the sampling area in the Fourier space is strongly deformed especially in cases of large rotation angle, as shown in Fig. 6. Therefore, it is very important to optimize the position and size of sampling window. In this study, as in Fig. 6(a), we set the representative points in the initial sampling area of Fourier space, whose shape and position is given by FFT, and traced the position of the sampling points through rotational transform. We determined the optimized sampling window from the minimum bounding box including the 50×50 representative points as in (b).

IV. Generation of High-Definition CGH for Comparison with Silhouettes Method

To proof the validity of the proposed method, the field of the 3D scene shown in Fig. 7 was calculated by using the proposed method. This 3D scene includes a torus object. Number of the samplings and the sampling interval of the calculated field are 65,536×65,536 and 1 µm, respectively. The torus is composed of 140 polygons. Fig. 8 shows the numerical reconstruction of the field by virtual optics using numerical lens. We can verify that the leakage light is reduced in the proposed method.

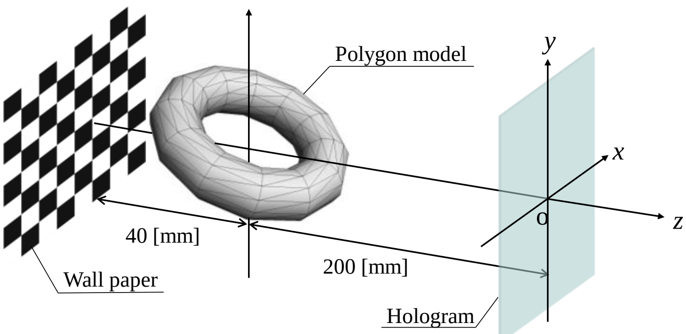

Figure 7. The 3D scene used for the examination of light-shielding.

V. Conclusion

We proposed a new technique to apply the rigorous light-shielding technique to high-definition computer holography. It is verified that the leakage light and occlusion error are reduced in the proposed method.

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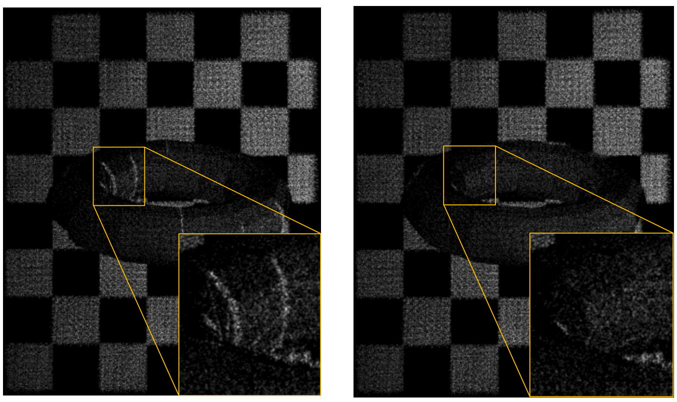

(a) Conventional silhouette method (b) This work

Figure 8. Comparison of two light-shielding techniques.