[Citation] International Display Workshop 2024 (IDW'24), Sapporo, paper 3Dp2-2, 787-790 (2024.12.4).

# Full-color full-parallax high-definition CGH with large background created using single FFT

**Takeshi Imai[1], Hirohito Nishi[1], Kyoji Matsushima[1]**

k821541@kansai-u.ac.jp (T. Imai)
[1] Department of Electrical, Electronic and Information Engineering, Kansai University
Yamate-cho 3-3-35, Suita, Osaka 564-8680, Japan.
Keywords: computer holography, full-color 3D image, FFT

**ABSTRACT**

*A novel method for computing a large background image in a short time is presented in full-color full-parallax high-definition computer-generated holograms. The proposed method uses a property of numerical wavefield calculation of phase-randomized images with single FFT. An actual full-color CGH is demonstrated.*

## 1 Introduction

Holography is a technology that can reconstruct impressively deep 3D scenes with natural motion parallax without any sensory conflict. Recent progresses in computer holography enabled us to create full-parallax high-definition computer-generated holograms (FPHD-CGH) with 0.36 trillion pixels [1,2], whose dimension is 34 cm × 34 cm and viewing angle is more than 45°. The gigantic number of pixels is due to the space-bandwidth product problem in computer holography. The viewing angle of a CGH increases almost inversely proportional to the pixel pitch of the fringe pattern, while the size of CGH is given by the product of pixel pitch and the number of pixels. Therefore, creating CGHs with both large size and viewing angle requires extensive number of pixels. As a result, creating FPHD-CGHs commonly requires very long computation time and large memory, and the reconstructed images are usually smaller than the size of CGH. This means that the 3D image has no large background filling the entire field of view (FOV) and the objects appear to be floating in outer space, as shown in Fig. 1 [3].

To address this issue, we have proposed a method for calculating a large monochromatic background image using single fast Fourier transform (FFT) [4]. This method utilizes change in sampling intervals after FFT for wavefield calculation of phase-randomized images and allows us to compute the wavefield of a very large background image without increasing the number of sample points. Therefore, required memory and computation time can be reduced significantly in this technique. However, the method cannot be applied to computation of full-color CGHs directly because the change in sampling intervals depends on the wavelength. As a result, the apparent size of the background image changes for each color channel. This effect causes severe color misalignments in the background image.

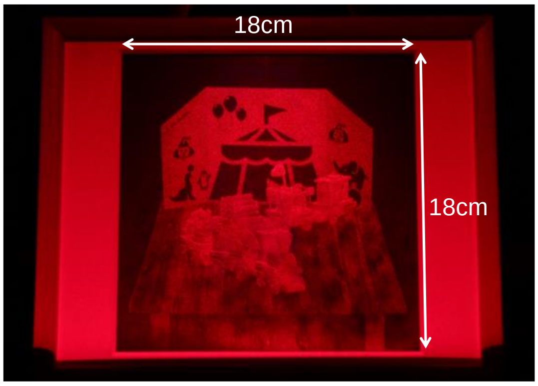


**Fig. 1 An example of optical reconstruction of FPHD-CGH. The total number of pixels is 100 G Pix.**

In this study, we propose an improved method to compute full-color FPHD-CGHs with a large background image. An actual full-color FPHD-CGH created by the proposed method is presented in this paper.

## 2 Principle

### 2.1 Calculation of a large background image

Figure 2 illustrates the principle of calculating wavefield with a large background image using single FFT. In this method, we assume that each pixel of the background image emits a spherical wave and the background wavefield is calculated by superimposing the spherical waves in the hologram plane. Here, note that although following discussion considers mainly the $x$-direction for simplicity, the same argument can be applied to the $y$-direction as well. The spherical wave emitted from pixel $m$ of the background image is given in the sample point $p$ in the hologram plane as follows:

$$f_m(x_p) = \sqrt{I_m}\exp[i(kr + \phi_m)] \tag{1}$$

where $I_m$ and $\phi_m$ are the brightness of the pixel $m$ and randomized phase, respectively. $r$ is the distance between pixel $m$ and sample point $p$ as follows:

$$r = \sqrt{(x_p - x_m)^2 + d^2} \tag{2}$$

where $d$ is the distance between the background image and hologram, and $x_p$ and $x_m$ are the position of the sample point $p$ and pixel $m$, respectively. In case of $d \gg (x_p - x_m)$, Eq. (2) can be approximated by

$$r \cong d + \frac{(x_p - x_m)^2}{2d} \,. \tag{3}$$

The background field in the hologram plane is given by superimposing all spherical waves emitted from each pixel of the background image as follows:

$$f(x_p) = \sum_{m=0}^{M-1} a_m \exp[ikr] \,, (p = 0, \dots, M-1) \,, \tag{4}$$

$$a_m = \sqrt{I_m} \exp[i\phi_m], \tag{5}$$

where $M$ is the number of sample points of the background image in the $x$-direction. Substituting Eq. (3) into Eq. (4), the background field is rewritten as

$$f(x_p) = \exp[ikd] \sum_{m=0}^{M-1} a_m \exp\left[i\frac{\pi}{\lambda d}(x_p - x_m)^2\right]. \tag{6}$$

Furthermore, when $x_p = p\Delta x$ and $x_m = m\Delta x_s$, Eq. (6) is rewritten as

$$f(x_p) \cong \exp[ikd] \exp\left[i\frac{\pi}{\lambda d}x_p^2\right] \times \sum_{m=0}^{M-1} a_m \exp\left[i\frac{\pi}{\lambda d}x_m^2\right] \exp\left[-i2\pi\frac{\Delta x \Delta x_s}{\lambda d}mp\right], \tag{7}$$

where $\Delta x$ and $\Delta x_s$ are a sampling interval of the hologram and pixel pitch of the background image, respectively. Here, by imposing the following constraint to Eq. (7):

$$\frac{\Delta x \Delta x_s}{\lambda d} = \frac{1}{M} \,. \tag{8}$$

Eq. (7) is represented using FFT as follows:

$$f(x_p) = \exp[ikd] \exp\left[i\frac{\pi \Delta x^2}{\lambda d}p^2\right] \mathrm{FFT}\left\{a_m \exp\left[i\frac{\pi \Delta x_s^2}{\lambda d}m^2\right]\right\}. \tag{9}$$

In this case, according to Eq. (8), the background image has apparent pixel pitches:

$$\Delta x_s = \frac{\lambda d}{M\Delta x}, \qquad \Delta y_s = \frac{\lambda d}{N\Delta y}, \tag{10}$$

where $N$ and $\Delta y$ are the number of sample points and sampling interval of the hologram in the $y$-direction, respectively. As a result, the apparent size of the background image is given by

$$W_{sx} = M\Delta x_s = \frac{\lambda d}{\Delta x}, \qquad W_{sy} = N\Delta y_s = \frac{\lambda d}{\Delta y}. \tag{11}$$

Because the apparent size depends on distance $d$, it can be larger than the size of the hologram, as shown in Fig. 3. Accordingly, we can calculate a large background image using single FFT in a short time.

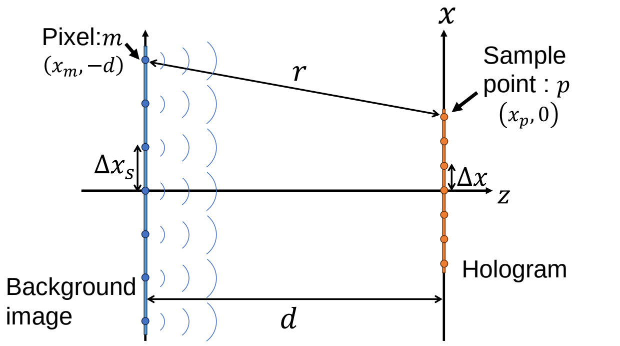


**Fig. 2 The principle of calculating a large background image using single FFT.**

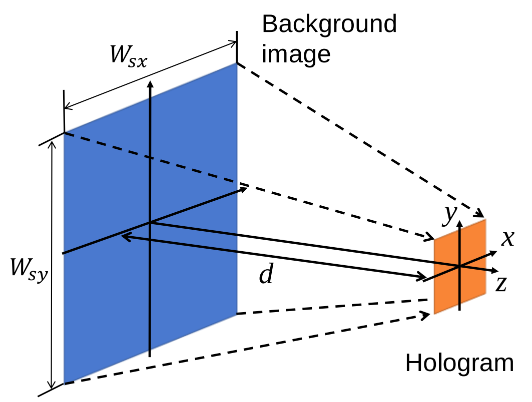


**Fig. 3 The apparent size of background image.**

## 2.2 Calculation for full-color CGH

When we calculate the wavefield of a full-color background image shown in Fig.4(a) using the conventional method, because the apparent size of the background image varies dependently on the red-green-blue (RGB) color channels, severe color misalignments are caused in the full-color reconstruction. Figure 4(b) shows the simulated reconstruction of the wavefield of the full-color background image calculated using Eq. (9). Here, virtual image formation is used for the simulated reconstruction [5].

To prevent the problem, the apparent size of the background image must not change for all color channels. Therefore, in this study, when calculating wavefields at the red and green wavelengths, the background images are trimmed so that the apparent sizes agree with that of the blue image, as shown in Fig. 5. In this case, because the appropriate size in the red and green images is smaller than the size of the sampling window, zeros are padded around the trimmed images. As a result, as shown in Fig. 4(c), a large full-

(a)

(b)

(c)

**Fig.4 Comparison of color background images by simulated reconstruction: (a) original image, (b) conventional method, and (c) proposed method.**

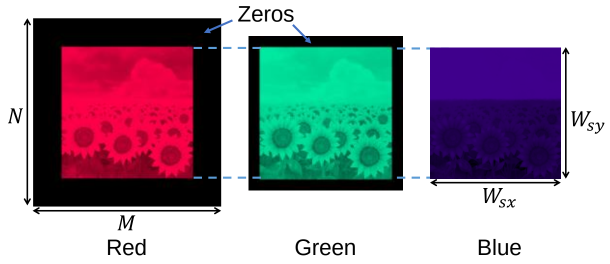


**Fig. 5 Resampling and zero-padding for avoiding color misalignments.**

color background image is successfully calculated without any color smear.

### 2.3 Occlusion processing

In the proposed method, because the background wavefield is directly calculated in the hologram plane, the mask-based occlusion processing that is a very powerful technique in creating FPHD-CGHs does not work well [6]. Thus, we performed the occlusion processing by the following procedure.

First, the background wavefield is calculated in the hologram plane using the proposed method. Then, the sampling window of the hologram plane is expanded to cover the entire field of view in the object plane. After that, the background wavefield is propagated back to the object plane using the band-limited angular spectrum method [7]. In the object plane, the occlusion processing of the background wavefield is conducted using the switchback technique [6]. Then the object field is added to the masked background field using the polygon-based method [8]. Finally, the wavefield in the object plane is again propagated to the hologram plane and trimmed to fit the hologram size.

## 3 Creation of full-color CGH

We fabricated a full-color FPHD-CGH using the proposed method and the technique based on RGB color filters [9]. Parameters of the CGH and specifications of the computer used for calculation are shown in Table 1 and 2, respectively. The total number of pixels is 34 billion pixels and viewing angle is 32° at blue color in this CGH. The 3D scene and photographs of optical reconstruction are shown in Fig. 6 and 7, respectively. The total computation time was approximately 45 hours.

We also calculated the same 3D scene without the proposed method to compare computation time and maximum memory usage. In this case, calculating a large background image requires a numerous number of sample points because we need a very large sampling window. However, because the memory capacity shown in Table 2 is insufficient for the large sampling window, the background image was divided into 3×3 segments and propagated to the object plane using the shifted angular spectrum method [10].

As a result, calculation only for the large background

**Table 1 Parameters of the created FPHD-CGH.**

| | |
|---|---|
| Number of pixels | 131,072 × 262,144 |
| Pixel Pitches [μm] | 0.8 × 0.4 |
| Design wavelengths (R,G,B) [nm] | (635, 517, 443) |

**Table 2 Specifications of the computer used for calculation.**

| | |
|---|---|
| CPU | Intel(R) Xeon(R) Gold 6342 |
| No. of cores | 96 |
| Memory [TB] | 2 |

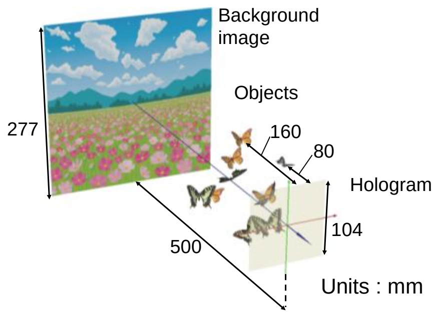


**Fig. 6 The 3D scene of the created FPHD-CGH.**

image is about 12.4 times faster than the case without the proposed method. However, the total speed-up rate was only 1.7. This is because calculation other than the large background image was the same as the conventional method. On the other hand, the maximum memory usage of the proposed method was approximately 644 GB. This is about half the value of the conventional method.

## 4 Conclusion

We proposed a method to calculate full-color FPHD-CGHs with a large background image using single FFT. The optical reconstruction of the fabricated FPHD-CGH shows the proposed method successfully calculated the wavefield of the large background image. The speed-up rate for the background image calculation was 12.4 in the FPHD-CGH. Our future work is to create a larger-scale FPHD-CGH that reconstructs an impressive scene, such as "sky".

This work was supported by Japan Society for the Promotion of Science (JSPS) KAKENHI 22H03712 and the commissioned research (JPJ012368C06801) by National Institute of Information and Communications Technology (NICT), Japan.

### References

[1] K. Matsushima and H. Nishi, “Challenges to tera-pixel-scale full-parallax computer holography for 3D Imaging,” in Frontiers in Optics + Laser Science (FiO, LS), FM5E, (2022).

[2] K. Matsushima, H. Nishi, R. Katsura, and C.-J. Lee,

"Computation techniques in tera-pixel-scale full-parallax computer holography for 3D display," The 12th Laser Display and Lighting Conference 2023, LDC10-02 (2023).

[3] K. Matsushima, Introduction to Computer Holography, Sect. 1.3 (Springer, 2020).

[4] H. Nishi and K. Matsushima, "Calculation technique for large backdrops filing visual field of full-parallax high-definition CGH," 12th Intern. Symp. on Display Holography, ISDH2023-ED-0256, 220-223 (2023).

[5] K. Matsushima, Introduction to Computer Holography, Sect. 13.3 and 13.5 (Springer, 2020).

[6] K. Matsushima, M. Nakamura, and S. Nakahara, "Silhouette method for hidden surface removal in computer holography and its acceleration using the switch-back technique," Opt. Express **22**, 24450-24465(2014).

[7] K. Matsushima and T. Shimobaba, "Band-limited angular spectrum method for numerical simulation of free-space propagation in far and near fields," Opt. Express **17**, 19662-19673 (2009).

[8] K. Matsushima and S. Nakahara, "Extremely high-definition full-parallax computer-generated hologram created by the polygon-based method," Appl. Opt. Vol. **48**, pp. 54–63 (2009).

[9] Y. Tsuchiyama and K. Matsushima, "Full-color large-scaled computer-generated holograms using RGB color filters," Opt. Express **25**, 2016-2030 (2017).

[10] K. Matsushima, Introduction to Computer Holography, Chap. 12, (Springer, 2020).

(a) High

(b) Left

(c) Center

(d) Right

(e) Low

**Fig. 7 Optical reconstruction of the created FPHD-CGH.**